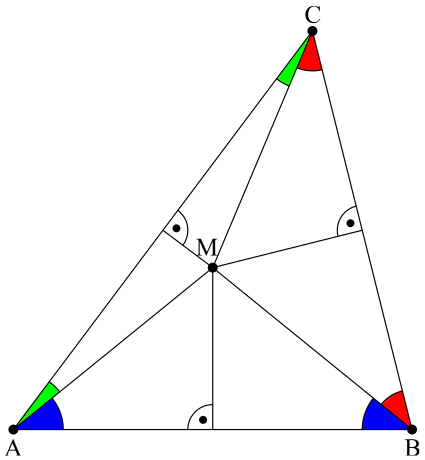
Jörg MEYER, Hameln

29. 10. 2021

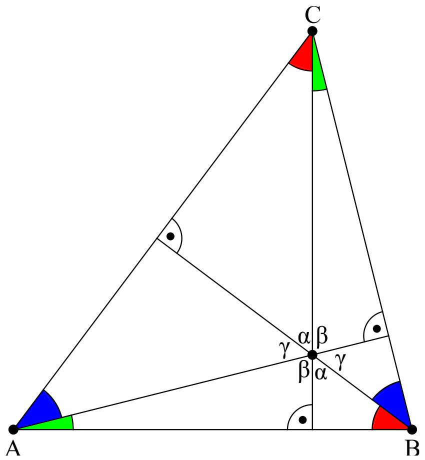
Differenzen von Dreieckswinkeln

Es sei ABC spitzwinklig.



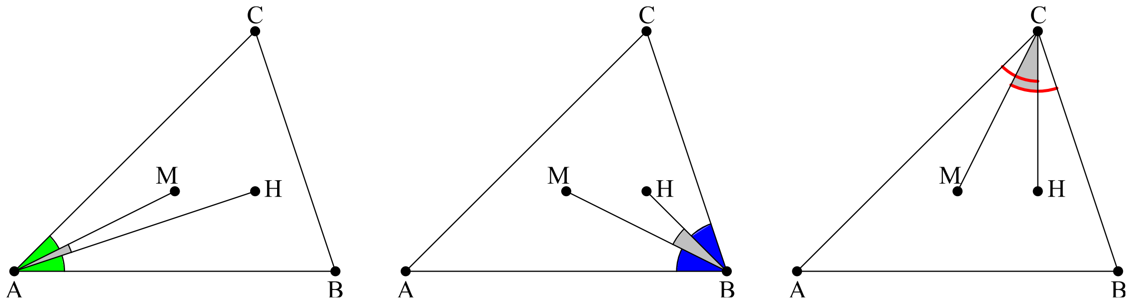
Der Schnittpunkt M der Mittelsenkrechten führt zu folgender Winkelkonstellation:

$$\begin{aligned} 90° &= \text{rot} + \text{blau} + \text{grün} \\ &= \text{grün} + \beta \\ &= \text{blau} + \gamma \\ &= \text{rot} + \alpha \end{aligned}$$



Der Schnittpunkt H der Höhen führt zu folgender Winkelkonstellation:

$$\begin{aligned} \text{gruen} &= 90° - \beta \\ \text{blau} &= 90° - \gamma \\ \text{rot} &= 90° - \alpha \end{aligned}$$



Links: $\text{grau} = \sphericalangle(\text{M, A, H}) = \alpha - 2 \cdot \text{grün} = \alpha - 2 \cdot (90° - \beta) = \alpha - \alpha - \beta - \gamma + 2 \cdot \beta = \boxed{\beta - \gamma = \sphericalangle(\text{M, A, H})}$.

Mitte: $\text{grau} = \sphericalangle(\text{M, B, H}) = \beta - 2 \cdot \text{blau} = \beta - 2 \cdot (90° - \gamma) = \beta - \alpha - \beta - \gamma + 2 \cdot \gamma = \boxed{\gamma - \alpha = \sphericalangle(\text{M, B, H})}$.

Rechts: $\text{grau} = \sphericalangle(\text{H, C, M}) = 2 \cdot \text{rot} - \gamma = 2 \cdot (90° - \alpha) - \gamma = \alpha + \beta + \gamma - 2 \cdot \alpha - \gamma = \beta - \alpha$, also ist rechts $\sphericalangle(\text{M, C, H}) = 360° - \sphericalangle(\text{H, C, M}) = \boxed{\alpha - \beta = \sphericalangle(\text{M, C, H})}$.